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6th International Meeting on Statistical Climatology

19 - 23 June, 1995

Galway, Ireland

6th International Meeting on Statistical Climatology "6 IMSC" 19-23 June, 1995 Galway, Ireland.

ON THE EFFECTS OF NONLINEARITY ON THE VARIABILITY OF LONG CLIMATIC RECORDS

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ABSTRACT

The largest monthly windrun series in Spain has been studied from a spectral point of view. After applying high and band-pass filters some of the resulting significant frequencies may be identified as related to a known dynamic or radiative phenomenon, as the annual cycle, the luni-solar and solar cycles (Mn and Sc), the QBO and the 14-month atmospheric Chandler tide. Most of the remaining peaks can be interpreted recalling the fact that combination tones are generated in nonlinear systems. The time evolution of the signals can be understood from this same point of view.

1 INTRODUCTION

A common spectral aproach to the study of climatic variability is carried out to assess the influence of nonlinearities on the variability of climatological series. In studies of climatic variability it is always desirable that samples are as large in time as possible. Firstly, this allows obtaining more information about the long term behaviour of the record, which is particularly important in trend studies. Secondly, it improves the reliability in signal detection. This is a consecuence of the fact that the signal-to-noise ratio increases proportionally to the number of samples in the series (Scargle, 1982). In this paper, the largest series of monthly windrun in Spain has been used; windrun is a very noisy variable which shows great variability in all timescales.

One problem with signal detection is associating the significant peaks obtained in the analysis with physical phenomena. Though significant fluctuations can be found in a great variety of frequencies, in many cases they are considered as more of a random than a physically fundamented nature. Herein, we provide a simple way in which some of the detected signals may be identified based on the fact that combination tones are expected to be generated in nonlinear physical systems. This interpretation has been used before in atmosphere applied studies by Hameed and Currie (1989) and Currie and Hameed (1990) to identify peaks detected in GCM simulated data spectra. We find that nonlinear interactions also seem to explain adequately the long term amplitude behaviour of the detected signals and consequently of the variance of the series.

In section 2 a brief description of data is provided. An overall view of the effects of nonlinearities on a physicall system is supplied in section 3. In section 4 results of filtering and spectrum analysis are provided together with a discussion following the guidelines proposed in section 3.

2 DATA

Data consist of monthly mean values of daily windrun records. There are seven sites in Spain which have available windrun data for a period of at least one hundred years (a reasonable length to attempt signal detection with simple methods). The series which has been used in this work is the largest of them all (1867-1992) and was built using data provided by two observatories sited in Madrid (Valero et al., 1995).

3 NONLINEAR INTERACTIONS

Herein, some possible nonlinear effects are shown: i) generation of harmonics; ii) generation of tones (sum and difference of frequencies); iii) amplitude modulation of waves; and iv) variance modulation of the series.

Following Currie and Hameed (1990) let us take the response $x_{out}(t)$ to be related to the input $x_{in}(t)$ in a nonlinear system as it could be the case of the climatic system (ε small compared to unity) by

$$x_{out}(t)=K\left[x_{in}(t)+\varepsilon x_{in}^{2}(t)\right] \qquad (1)$$

where K is constant.

i) Harmonics generation

Let us consider an input function $x_{in}(t)=\cos\omega t$; there will be a component in the output, $x_{out}(t)$, given by

$$K\frac{\varepsilon}{2}\cos 2\omega t \qquad (2)$$

Therefore, if we consider a second order nonlinearity a harmonic term is generated in a frequency double of the original frequency. If the order of the nonlinear term is greater (say **n**) the generated harmonic would be a frequency n times that of the original frequency.

ii) Combination signals

Further, if we assume an input function

$$x_{in}(t) = A\cos\omega_1 t + B\cos\omega_2 t \quad (3)$$

where ω_1 and ω_2 are not in harmonic relation, apart from the harmonics generated according to (2), there will be a component in the output given by

$$2K\varepsilon AB\cos\omega_1 t\cos\omega_2 t \quad (4)$$

This term can be re-written into two components with frequencies corresponding to the sum and difference of the "parents" frequencies, namely

$$K\varepsilon AB\left[\cos(\omega_1+\omega_2)t+\cos(\omega_1-\omega_2)t\right] \quad (5)$$

Thus, considering the smallest order perturbation (second order) from linearity as in (1), two new signals of frequencies $\omega_1 \pm \omega_2$ should be expected to appear (sum and difference tones).

iii) Modulation of amplitude

When spectrum analysis is applied to a series of N data, values of espectral density will be evaluated for a set of N/2 frequencies separated by a frequency distance of 1/N. Therefore, for any frequency ω_2 lower than 1/N the possible sum and difference tones that would be produced by its interaction with ω_1 would be separated from ω_1 by less than one frequency interval. In those cases it is not possible to distinguish ω_1 from the sum and difference tones and expression (4) would then be interpreted as a single term: the modulation of the amplitude of a signal of frequency ω_1 by a sinusoidal function of frequency ω_2 ($f(B,\omega_2,t) = 2B\cos\omega_2 t$). Expression (4) can be re-written as

$$K\varepsilon Af(B,\omega_2,t)\cos\omega_1 t \quad (6)$$

Further, by substituting (3) in (1) and using (6), the whole contribution of ω_1 to the output would be given by

$$K\left\{A\left[1+\varepsilon f(B,\omega_2,t)\right]\right\}\cos\omega_1 t \quad (7)$$

iv) Variance modulation

The modulation stated in **iii)** for nonlinear systems can also affect the variance of the series. Let us consider a series x_t which can be expressed using Fourier analysis as

$$x_t = \bar{x} + \sum_{p=1}^{N/2}\left[R_p\cos(\omega_p t+\phi_p)\right] \quad (8)$$

where R_p and ϕ_p are the mean amplitude and phase of the p-th Fourier harmonic of frequency ω_p in the considered time period (t=1...N). According to Parseval's theorem (Chatfield, 1984) the variance of the series can be written as a function of the amplitudes R_p

$$\hat{\sigma}^2 = \sum_{p=1}^{N/2-1}\left[R_p^2/2\right]+R_{N/2}^2 \quad (9)$$

According to (7), in case a low frequency ω (<1/N) exists the amplitude of the p-th Fourier harmonic, R_p, of frequency ω_p would be given by the temporal average of a function $R'_p\left[1+\varepsilon f(R,\omega,t)\right]$ where R'_p (similarly to A in section **iii)**) is the amplitude of the ω_p-wave modulated by the ω-wave. Allowing that each one of the amplitudes may be modulated by ω, the variance of the series can also vary in time modulated by a function of frequency ω(*i.e.*, the amplitudes and the variance can have different values for different subperiods in the series).

4 RESULTS

Periodograms have been used as spectral estimators (Bloomfield, 1976). Though it is more noisy than other smoothed spectral estimators, it provides more resolution and allows the user to identify single peaks. Care must be taken in identifing this peaks and therefore it is essential making use of an appropiate powerful test. In this work, Fisher's test of significance (Fisher, 1929; Shimshoni, 1971) has been applied.

After applying the Fisher's test (99% significance) to the monthly windrun series eleven cycles and trend were found to be significant (Table 1; marked with *). Most of the variance is explained by the annual cycle and its six and four month harmonics (17.5%). There is also an important amount of variability produced by very low frequency oscillations (VLFO) of a period around 60 years. This cycle has been pointed out by Folland *et al.* (1984) and Schlesinger and Ramankutty (1994) and it is characterized by a deep minimum around the 1920's, a maximum around the 1950's followed by a downward trend which reaches a new minimun in the 1960's. Figure 1 shows a 4-years centered moving average of the original series. The above mentioned long term features are apparent. Luni-Solar cycle (Mn) and solar cycle (Sc) have also been detected, together with other signals of periods around 25 years, 9 years, 5 years and 10 months.

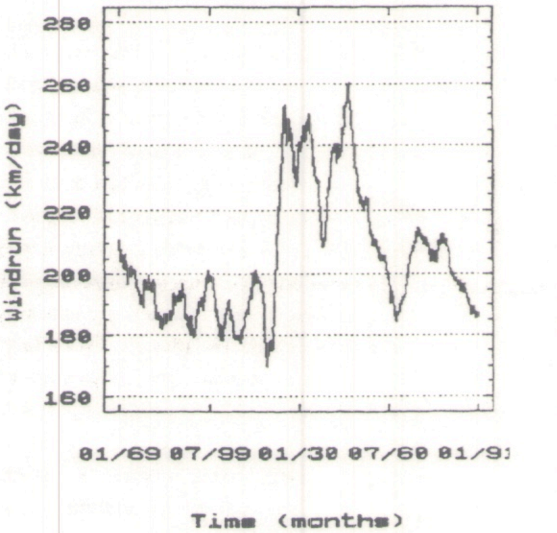

Figure 1. 48-month centered runing average of original series (x axis units are months/year)

High and band-pass filters have been applied to the data to eliminate the VLFO and the annual cycle and harmonics. The high-pass filter has been applied through subtraction of the low pass filter output to the data (Bloomfield 1976, p. 129; filter length n=180 and cutoff frequency, ω_c= 0.0035 cycles/month). The band-pass filter has been designed using complex demodulation (Bloomfield, 1976; n= 175 and ω_c= 0.0034 cycles/month). Finally, the 12, 6 and 4 month signals have been remodulated and subtracted to the series. Thirty five cycles which could be grouped and identified as contributions of several signals have been found to be significant after applying the same analysis as before (see Table 1). In particular, the QTO (Quasi-

Table 1. Frequencies, periods and possible associated fenomena of detected significant signals. * indicates signals detected before filtering. Values in brackets below associated phenomenon indicate estimated tone frequencies.

	FREQUENCY (cycles/month)	PERIOD (months or years)	ASSOCIAT. PHENOM.
1*	0.00066	126 yr	Trend
2*	0.00132 0.00198	63 yr 42 yr	VLFO
3*	0.00331	25.2 yr	I(Mn-VLFO)
4*	0.00397 0.00434 0.00520	21 yr 19.2 yr 16 yr	Mn
5*	0.00694 0.00727 0.00779	12 yr 11.45 yr 10.7 yr	Sc
6*	0.00868 0.00926 0.00958 0.01045	9.6 yr 9 yr 8.7 yr 8 yr	I(VLFO+Sc)
7	0.01388	6 yr	I(A12-Ch) (71.1 m)
8*	0.01595 0.01585 0.01650	5.6 yr 5.25 yr 5.1 yr	
9	0.02170	46.1 m	QTO
10	0.03831	26.1 m	QB0
11	0.04348	23.0 m	I(A12-QBO) (21.82 m)
12	0.06849 0.07042	14.6 m 14.2 m	Ch
13*	0.08333	12 m	A12
14*	0.09900	10.1 m	I(A12-QTO) (10.27 m)
15	0.07463	13.4 m	I(A12-Sc) (13.2 m)
16	0.13812	7.24 m	I(A6-QBO) (7.74 m)
17	0.14493	6.9 m	I(A6-QTO) (7.06 m)
18	0.15456	6.47 m	I(A6-QBO) (6.55 m)
19*	0.16777	6 m	A6
20	0.19608	5.1 m	I(A6+QTO) (5.22 m)
21	0.20408	4.9 m	I(A6+QBO) (4.9 m)
22	0.20492	4.88 m	I(A4-QBO) (4.71 m)
23*	0.25	4 m	A4
24	0.5	2 m	A2

Triennial Oscillation, influence of El Niño in the Atlantic Ocean) has been found with a period of 46.1 months. Significant evidence of the QBO (26.1 months), the 14-month Chandler wobble (Ch) and the 2-month harmonic of the annual cycle have also been found. The rest of the cycles cannot be directly associated to a dynamic or radiative physical phenomenon.

Nonlinear effects

Possible sum and difference tones were generated according to (5) using the annual cycle and its harmonics with the QBO, QTO and Ch-tyde as "parents" frequencies. The resulting estimated tones agreed well with some of the found frequencies. These generated signals are indicated in Table 1 by *I(A,B)* where *A* and *B* are the generating frequencies. Most of them were also detected by Currie and Hameed (1990) using model data. In Table 1 estimated tones are indicated in brackets below *I(A,B)*. Only in cases where *VLFO* is involved it is not possible to make an adecuate estimation due to the wide range of uncertainty in estimating its frequency.

The time evolution of the signals has been studied using moving periodograms (periodograms applied to 30 year data subsets 5-years shifted in time). All amplitudes show similar oscillations to the VLFO (see Figure 2) pointing out a modulation of by this signal according to (7).

The time evolution of the variance is described through calculating 48-month centered moving variances in a similar way to the usual centered moving averages. Figure 3 shows a plot of the resulting curve. Once again the resulting trends agree with the long term trends found for the original series (see Fig. 1). There are other low frequency signals also apparent in Figure 1 that also seem to drive the evolution of the variance. Anual oscillations (*i.e.*, annual maximum minus minimun) were also calculated providing the same results (not shown here).

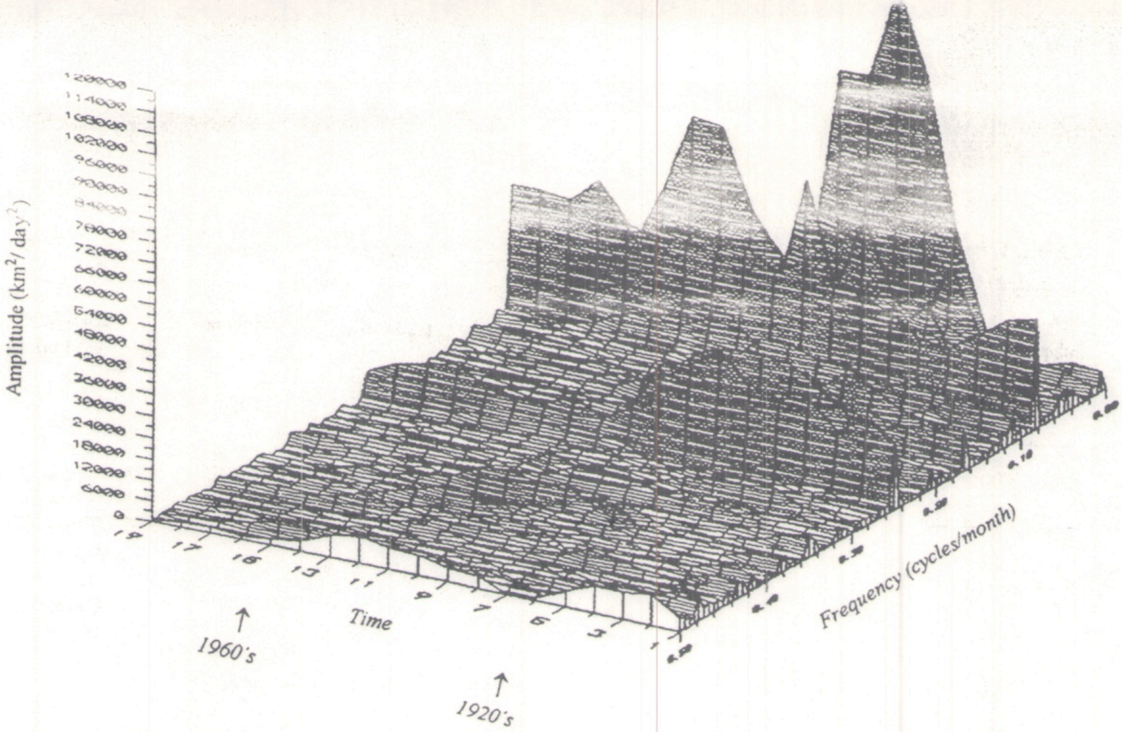

Figure 2. Moving periodograms of original series. Periodograms have been calculated for 30 year periods shifted 5 years in time.

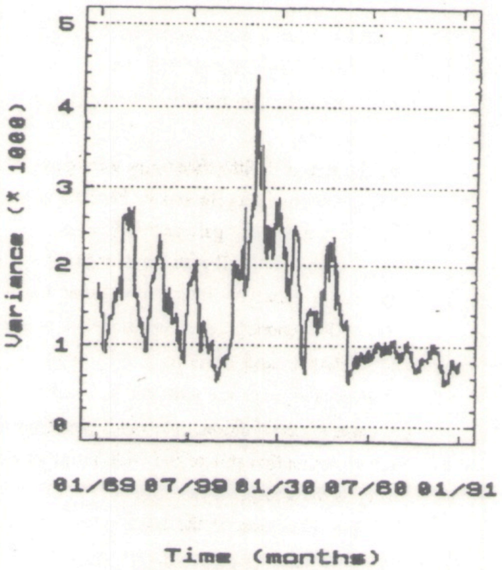

Figure 3. 48-month centered runing variance of original series (x axis units are months/year)

5 CONCLUSIONS

In this work an example is given of the effects of nonlinearities in the variability of a series, both in the generation of combination tones and in the modulation of the constituent signals and the variance of the series. The values expected according to a nonlinearity of second order agree quite well with the results obtained from direct meassurement. Also, the amplitudes of the constituent signals of the series seem to be modulated by a very low frequently oscillation that causes relative minima in the 1920´s and 1960´s and relative maxima in the 1940´s and at the end of the record. The time evolution of the variance of the series shows a similar pattern.

Recent calculations for windrun data from other sites are being carried out by the authors. Results seem to confirm on the whole the results presented here.

ACKNOWLEDGEMENTS

This work was supported by the Comunidad Autonoma de Madrid (Spain), under Contract CAM 286/92 and grants 316/92.

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6 IMSC

Sixth International Meeting on Statistical Climatology

Galway, Ireland: June 19 -23, 1995

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Mr. J. Fidel Gonzalez,
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Dear Mr. Gonzalez,

I acknowledge with thanks receipt of your registration form for the Sixth International Meeting on Statistical Climatology (6IMSC), together with payment of IR£140.00.

Following your instructions, accommodation has not been reserved for you.

If you have any queries in relation to any aspect of 6IMSC, please do not hesitate to ask (preferably by either fax or e-mail). I look forward to welcoming you to Ireland in June, and hope that 6IMSC will prove a professionally rewarding and socially enjoyable experience.

Yours sincerely,

Iognáid G. Ó Muircheartaigh.
Program Chair, 6IMSC.